1. Overview

The Non-Rigid Structure from Motion (NRSFM) problem: given corresponding 2D points in multiple images of a non-rigid object, the goal is to recover the object's 3D shape and pose in each image.

Standard approach: low-rank matrix factorization, with 3D shapes modeled within a low-dimensional linear space; non-linear deformations weaken the low-rank constraint, increasing the number of basis shapes in the linear model.

Kernel NRSFM: using the kernel trick, our new model complements the low-rank constraint by capturing non-linear relationships in the coefficients of the standard model; our model is flexible and can use different kernel functions.

2. NRSFM

Matrix Factorization: a linear space with $K$ basis shapes $\{S_k\}$.

\[ W = DF + S \]

Input

<table>
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<tr>
<th>observed 2D shapes</th>
<th>camera matrices</th>
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Output

| 3D shape of $i^{th}$ image: $S(c_i) = \sum_{k=1}^{K} c_{i,k} S_k$ |

Low-rank constraint: with $K$ basis shapes, $\text{rank}(W) \leq 3K$.

The Artificial Slinky Toy: 1 degree of freedom:

3. Kernel SRM

General approach: for each observed 2D shape in $W$, the reconstructed 3D shape is the output of a non-linear mapping $(\leftrightarrow)$ of a point $c_i$ in an $h$-dimensional shape space ($h = 2$).

\[ W = \text{D}(K_{c,b} K_{b,c}^{-1} K_{b,c}) \]

\[ W = \text{D}(\text{CC}^T \otimes \text{I}_3) D^T \]

Use generalized inner products given by a kernel function, e.g.,

\[ \kappa(c_i, c_{i'}) = \sigma(c_i - c_{i'}) \]

**Problem:** low-rank constraint is lost because $K$ is full rank!

**Solution:** consider a subset of basis shapes, $\{b_1, \ldots, b_K\}$.

**The Low-Rank Kernel Trick:**

\[ K \approx K_{c,b} K_{b,c} K_{b,c}^{-1} K_{b,c} \]

The non-unique factorization of $W$ is then obtained with

\[ M = \text{D}(K_{c,b} K_{b,c}^{-1} \otimes \text{I}_3) \]

The 3D shape reconstructed for the $i^{th}$ image is

\[ S(c_i) = \{ \kappa(c_i, b_1) \ldots \kappa(c_i, b_K) \otimes \text{I}_3 \} \text{M}^T W \]

with $c_i, b_k \in \mathbb{R}^h$ ($h < K$) in a more compact shape space!

**Kernel Shape Trajectory Approach (KSTA):**

Compact representation $X$ of smooth shape deformation [1]:

\[ x = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \]

\[ b_k^T = \cos(t_k)^T X \]

single scalar $t_k$ defines $b_k$ along the shape trajectory

4. Experimental Results

Validation on motion capture and artificial 3D datasets:

- **Learned Models:** shape spaces and non-linear mappings,

- **Quantitative Evaluation:** normalized reconstruction errors,

- **References and Acknowledgements**


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